

# MODELING OF GENERALIZED DOUBLE RIDGE WAVEGUIDE T-JUNCTIONS

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## ABSTRACT

Eigen modes in double ridge waveguide are obtained using mode-matching technique, and the scattering parameters of generalized E-plane double ridge T-junctions are computed using cascading procedure and improved three plane mode matching method [7]. The accuracy and versatility of the method are verified through several numerical examples. The computed results are in good agreement with the measured results and that by other approach.

## I. INTRODUCTION

Waveguide T-junctions are important components in modern communication systems and many other microwave applications [1-4][9]. Because ridge waveguides have low cutoff frequency and wide bandwidth [10][11], a broad band ridge waveguide T-junction was proposed for wide band applications [6][8]. Although few types of single ridge T-junctions have been studied, the perpendicular arm of the T-junction has to be empty waveguide [6], or the two main arms have to be identical [8]. In addition, only dominant mode's scattering parameters were obtained. Because of these limitations, the studied structures are limited in their use, and accurate analysis and design of the generalized ridge waveguide T-junctions become necessary.

The complexity of the ridge waveguide T-junction makes its direct analysis very difficult. Three plane mode-matching technique (TPMMT) [5][7], in which one port of the T-junction is shorted, and only two-port network's scattering parameters are computed, provides a way to greatly simplify and solve the complicated T-junction problems.

In this paper, modes in double ridge waveguide are computed by the mode matching technique. Then the three plane mode-matching and a cascading procedure

are used to model the generalized double ridge waveguide T-junctions. The perpendicular arm of the ridge T-junction is shorted, and can be viewed as two sections of double ridge waveguides cascaded with a single ridge waveguide section. Solving the discontinuity between two ridge waveguides and using cascading procedure, the generalized scattering matrices of the two port network can be obtained. By changing the short position three times, the generalized S-parameters are computed for all modes in the two horizontal arms and only the dominant mode in the perpendicular arm of the ridge T-junctions. The numerical results of the asymmetrical waveguide and single ridge T-junctions are compared with the measured results and those of previous work, and it is shown that they are in good agreement. Finally the S-parameters of a double ridge waveguide T-junction are computed and presented.

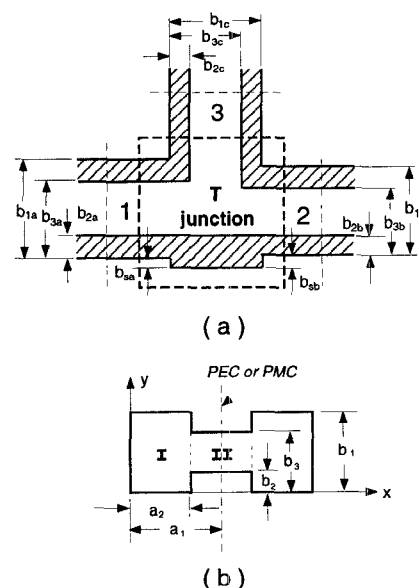


Fig. 1 Generalized double ridge T-junction (a) Sectional side view of the T-junction (b) Cross section of each arm.

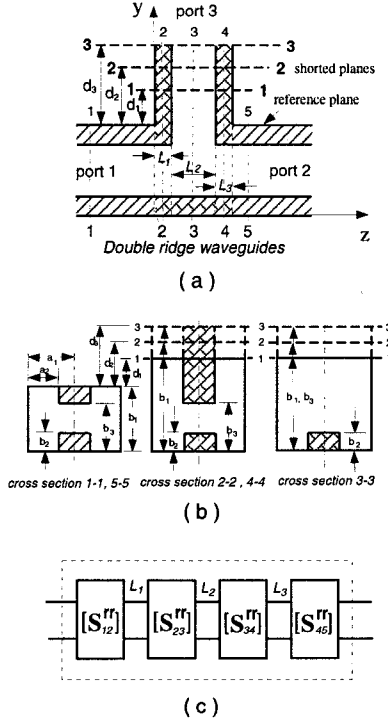


Fig. 2 (a) Side view of the perpendicular arm shorted double ridge T-junction (b) Cross section of port 1 and equivalent ridge waveguides (c) Network representation of the double ridge T-junction.

## II. METHOD OF ANALYSIS

The double ridge waveguide T-junction under consideration is shown in Fig. 1. The cross sections of the ridge waveguides are symmetric as shown in Fig. 1(b). By putting a perfect electric conductor (PEC) or a perfect magnetic conductor (PMC) at the symmetrical planes, only half structure needs to be considered. Before solving the double ridge to double ridge discontinuity and applying cascading procedure, eigen modes in each ridge waveguide have to be found first.

### A. Modes in Double Ridge Waveguide

A double ridge waveguide is divided into two regions, region I and region II, as shown in Fig. 1(b). The fields of the eigenmodes are expanded by normal TE and TM modes in region I and region II. By forcing the tangential electric and magnetic fields at the boundary to be continuous, and taking proper inner product, the characteristic equation for the cutoff frequencies can be obtained.

The cutoff frequency of the double ridge waveguide can be found by searching for the zeros of the determinant of the characteristic matrix. The field coefficients of the eigenmodes are then obtained by solving

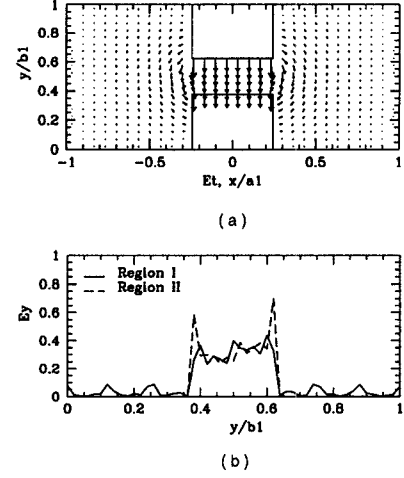


Fig. 3 Field distribution obtained from Mode Matching with  $a_1 = 0.45''$ ,  $a_2 = 0.34''$ ,  $b_1 = 0.40''$ ,  $b_2 = 0.15''$ ,  $b_3 = 0.25''$ . (a) 2-D tangential field plot (b)  $E_y$  at boundary between region I and region II.

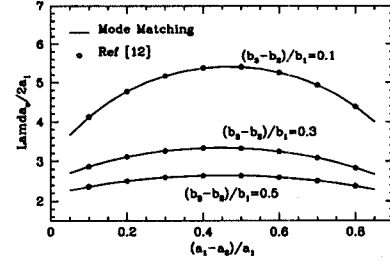


Fig. 4 Cutoff wavelength of a double ridge waveguide with aspect ratio  $b_1/a_1 = 0.5$ .

the linear characteristic matrix. Only pure TE and TM modes exist in the ridge waveguide. Empty waveguide and single ridge waveguide are the special cases of the double ridge waveguide.

### B. Double Ridge to Double Ridge WG Discontinuity

When the perpendicular port is shorted, the resulting two port network can be treated as several double ridge waveguides with certain lengths connected together as shown in Fig. 2. A full wave mode-matching technique is used to obtain the generalized scattering matrices of double ridge to double ridge discontinuities. The fields in two ridge waveguides are the summation of incident and reflected waves of all ridge waveguide's eigen modes. By applying boundary condition at the interface of two ridge waveguides, and taking proper inner product, following equation can be obtained:

$$\begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix} = \begin{bmatrix} [S_{11}] & [S_{12}] \\ [S_{21}] & [S_{22}] \end{bmatrix} \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{bmatrix} = [S] \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{bmatrix} \quad (1)$$

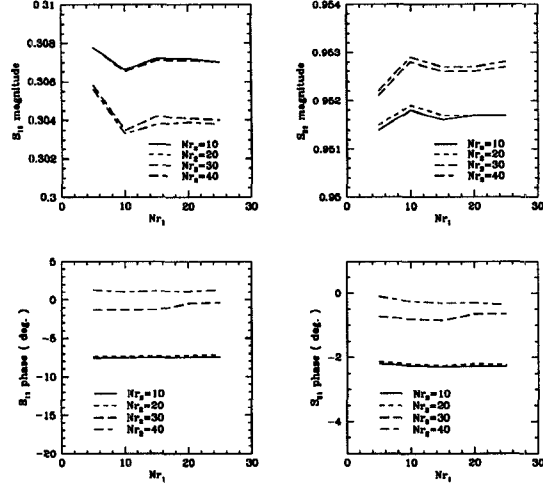


Fig. 5 Convergence of S-parameters with  $N_{r1}$  and  $N_{r2}$  (number of modes in each region) of two double ridge waveguides.  $a_1 = 0.45''$ ,  $a_2 = 0.34''$ ,  $b_s = 0.05''$ .  $b_1 = 0.40''$ ,  $b_2 = 0.15''$ ,  $b_3 = 0.25''$  of ridge WG 1 and  $b_1 = 0.50''$ ,  $b_2 = 0.15''$ ,  $b_3 = 0.35''$  of ridge WG 2.

where  $b_1$  and  $a_1$  are vectors of size  $N_1 = N_1^h + N_1^e$ , representing the reflected and incident mode coefficients in ridge waveguide 1,  $b_2$  and  $a_2$  are vectors of size  $N_2 = N_2^h + N_2^e$ , ( $N^h$  is the number of  $TE$  modes, and  $N^e$  is the number of  $TM$  modes used in the computation), representing the mode coefficients in ridge waveguide 2, respectively.  $[S]$  is generalized S-matrix.

Having computed the S-matrix of two ridge waveguides  $[S_i]$ , S-matrix cascading procedure is used to characterize the two port double ridge waveguide network by taking into account the effect of the higher order mode's interactions. For the case of symmetrical T-junction, only half of the discontinuities is needed to be computed so that computation time can be reduced by a factor of 2.

### C. Extraction of S-matrix of the T-junction

It is assumed that the dominant mode is the only propagating mode in the perpendicular arm of the ridge T-junction and that higher order modes are negligible beyond the short circuit positions. The S-matrices of a ridge waveguide T-junction can be extracted from the three two-port generalized S-matrices  $[S^{t-t}]$  by putting a short plane at three different positions  $t-t$  ( $t = 1, 2, \text{ and } 3$ ) at the perpendicular arm of the ridge T-junction (Fig. 2) as [7]:

$$S_{33} = \frac{\Delta_{21} [S_{11}^{1-1}]_{11} + (\Delta_{12} - \Delta_{32}) [S_{11}^{2-2}]_{11} - \Delta_{23} [S_{11}^{3-3}]_{11}}{\Gamma_1 \Delta_{21} [S_{11}^{1-1}]_{11} + \Gamma_2 (\Delta_{12} - \Delta_{32}) [S_{11}^{2-2}]_{11} - \Gamma_3 \Delta_{23} [S_{11}^{3-3}]_{11}} \quad (2)$$

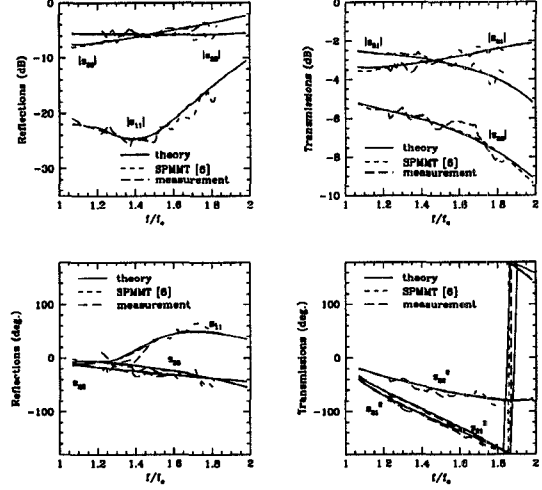


Fig. 6 S-parameters of an asymmetrical waveguide E-plane T-junction (all ridges are zero). (a) Amplitude (b) phase. T-junction dimensions  $a_{1a} = a_{1b} = a_{1c} = 0.45''$ ,  $b_{1a} = 0.40''$ ,  $b_{1b} = 0.21''$ ,  $b_{1c} = 0.275''$ ,  $b_{sa} = 0.0''$ ,  $b_{sb} = 0.19''$ .

$$[S_{ij}] \quad i \neq 3, j \neq 3 =$$

$$\Gamma_1 \Delta_{21} \left( \frac{1}{\Gamma_1} - S_{33} \right) [S_{ij}^{1-1}] + \Gamma_2 \Delta_{12} \left( \frac{1}{\Gamma_2} - S_{33} \right) [S_{ij}^{2-2}] \quad (3)$$

$$[S_{i3}] [S_{3j}] \quad i \neq 3, j \neq 3 =$$

$$\left( \frac{1}{\Gamma_1} - S_{33} \right) [S_{ij}^{1-1}] - \left( \frac{1}{\Gamma_1} - S_{33} \right) [S_{ij}] \quad (4)$$

where  $\Delta_{ij} = \frac{\Gamma_i}{\Gamma_i - \Gamma_j}$ , and  $\Gamma_i = -e^{-j2\beta d_i}$ .  $\beta$  is the propagation constant of the dominant mode in perpendicular arm.

## III. RESULTS

Computer programs have been developed to find the eigenmodes of the double ridge waveguide and compute the S-matrix of the double ridge waveguide T-junctions. The computed fields at the boundary of ridge waveguide's region I and II, and at the conducting surface of the ridge waveguide, shown in Fig.3, ensure the correctness of the eigenmode's field coefficients. Fig. 4 shows the normalized dominant (TE) mode cutoff wavelength of a double-ridged waveguide with aspect ratio  $b_1/a_1 = 0.5$ . The results are in good agreement with [12]. Intensive convergence tests have been performed. Fig. 5 presents the convergence of the dominant mode's S-parameters for a double ridge to double ridge discontinuity. For the ridge waveguide region with smaller cross section more than 20 modes are sufficient

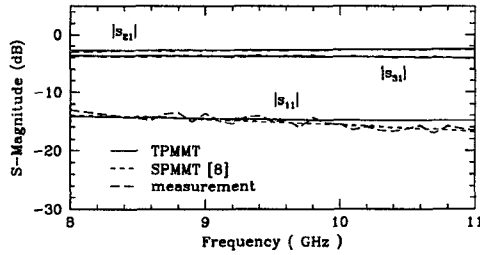


Fig. 7 S-parameters of a single ridge waveguide T-junction T-junction dimensions  $a_{1a} = a_{1b} = a_{1c} = 0.45''$ ,  $a_{2a} = a_{2b} = a_{2c} = 0.34''$ ,  $b_{1a} = b_{1b} = b_{1c} = 0.40''$ ,  $b_{2a} = b_{2b} = b_{2c} = 0.30''$ ,  $b_{3a} = b_{3b} = b_{3c} = 0.40''$ ,  $b_{sa} = b_{sb} = 0.0''$ .

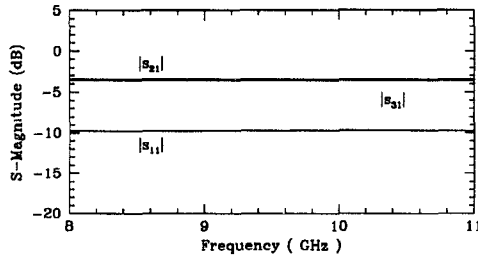


Fig. 8 S-parameters of a double ridge waveguide T-junction T-junction dimensions  $a_{1a} = a_{1b} = a_{1c} = 0.45''$ ,  $a_{2a} = a_{2b} = a_{2c} = 0.34''$ ,  $b_{1a} = b_{1b} = b_{1c} = 0.40''$ ,  $b_{2a} = b_{2b} = b_{2c} = 0.15''$ ,  $b_{3a} = b_{3b} = b_{3c} = 0.25''$ ,  $b_{sa} = b_{sb} = 0.0''$ .

for convergence in most cases studied, but more modes are needed for fields in the ridge waveguide region with larger cross section. It is found that, when three short plane positions are within  $0.6 b_1$  to  $3.0 b_1$  of port 1 or 2, both good results and efficiency (reasonable number of modes) can be achieved.

Fig. 6 shows the S-parameters of an asymmetrical waveguide E-plane T-junction (all the ridges are zero). It is shown that the computed results are in excellent agreement with the measured results and that calculated by SPMMT [6].

Fig. 7 shows the computed scattering parameters of a single ridge T-junction. The numerical results are also in good agreement with the measured results and that from SPMMT [8].

Finally, a double ridge waveguide T-junction's scattering parameters are computed and shown in Fig.8. It is shown that  $S_{31}$  is closer to  $S_{21}$  than that of single ridge T-junction.

#### IV. CONCLUSION

Generalized double ridge T-junction is modeled by cascading the double ridge discontinuity procedure and

the improved three short plane mode matching technique. Modes and field coefficients are obtained by mode matching technique. The accuracy and versatility of the method are verified through several numerical examples and compared with the measured results and that by another method.

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